HW 5, due today, Put in pile at front.
HW 5a, due never, but you need to know this material for midterm 2.

Midterm 2 is Wednesday.
The midterm covers chapter 3.

Entry Task:
Find the quasi-frequency and quasi-period of the mass-spring system modeled by

$$3u'' + 2u' + 4u = 0$$

3.7 and 3.8: Mass-Spring Systems

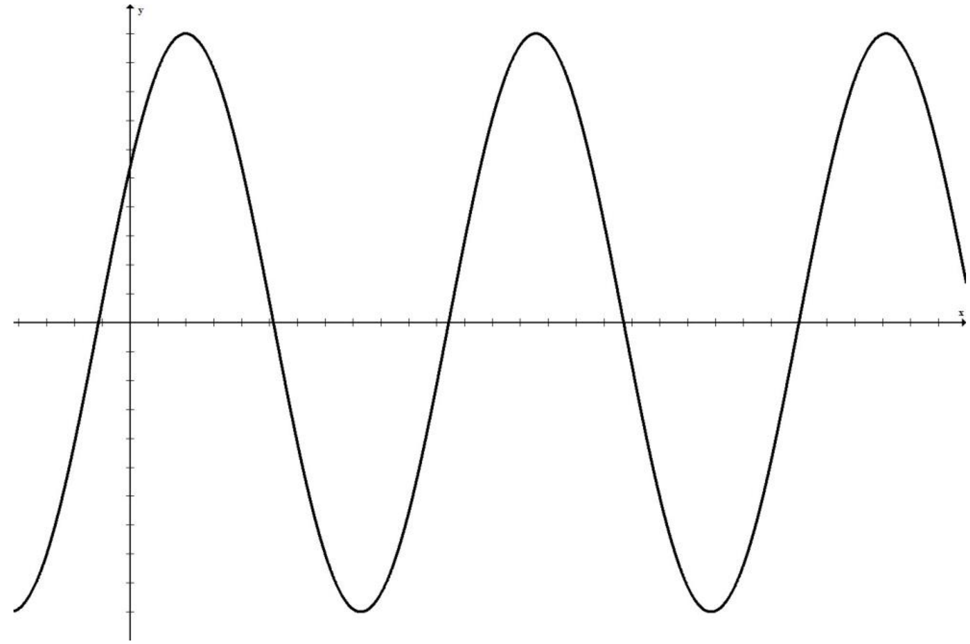
m = mass attached to end of spring
γ = damping constant
k = spring constant
F(t) = external force
u(t)= displacement from rest at time *t*

We derived that

$$mu'' + \gamma u' + ku = F(t)$$

3.7 Summary

F(t) = 0 (no external forcing).



***Case 1*: F(t) = 0 and $\boldsymbol{\gamma = 0}$**

$$mu'' + ku = 0$$

$$mr^2 + k = 0 \text{ gives } r = \pm\sqrt{k/m}\; i$$

Soln: $u(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$

$$\omega_0 = \sqrt{k/m} = \boldsymbol{natural}\ freq.$$

$$R = \sqrt{c_1^2 + c_2^2} = amplitude.$$

Case 2: **F(t) = 0 and $\gamma > 0$**

$$mu'' + \gamma u' + ku = 0$$

$$mr^2 + \gamma r + k = 0 \text{ gives}$$

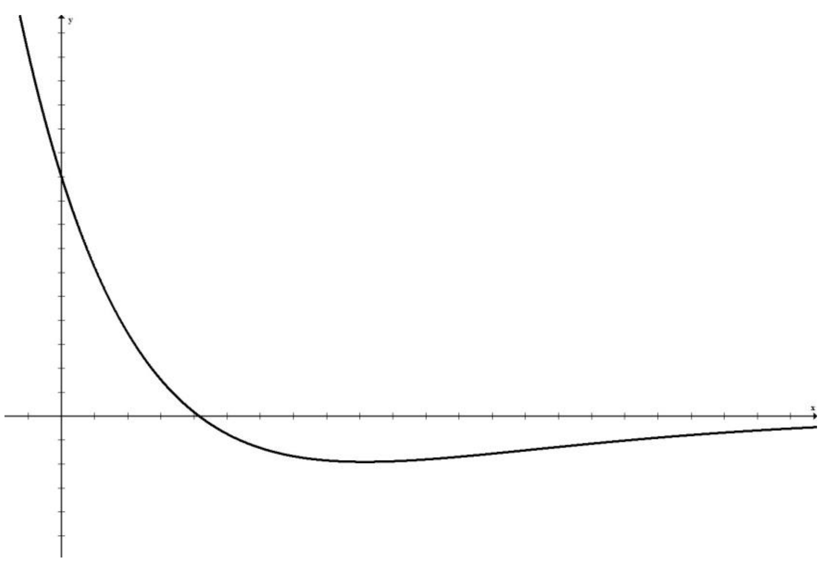
$$r = -\frac{\gamma}{2m} \pm \frac{1}{2m}\sqrt{\gamma^2 - 4mk}$$

2a: $\gamma > 2\sqrt{mk}$, **overdamped**

Soln: $u(t) = c_1 \mathrm{e}^{\mathrm{r}_1 t} + c_2 \mathrm{e}^{\mathrm{r}_2 t}$

2b: $\gamma = 2\sqrt{mk}$, **critically damped**

Soln: $u(t) = c_1 \mathrm{e}^{\mathrm{r}t} + c_2 t \mathrm{e}^{\mathrm{r}t}$



2c: $\gamma < 2\sqrt{mk}$, **damped vibrations**

$$r = -\frac{\gamma}{2m} \pm \frac{1}{2m}\sqrt{4mk - \gamma^2}\, i$$

Soln:$u(t) = e^{\lambda t}(c_1 \cos(\mu t) + c_2 \sin(\mu t))$

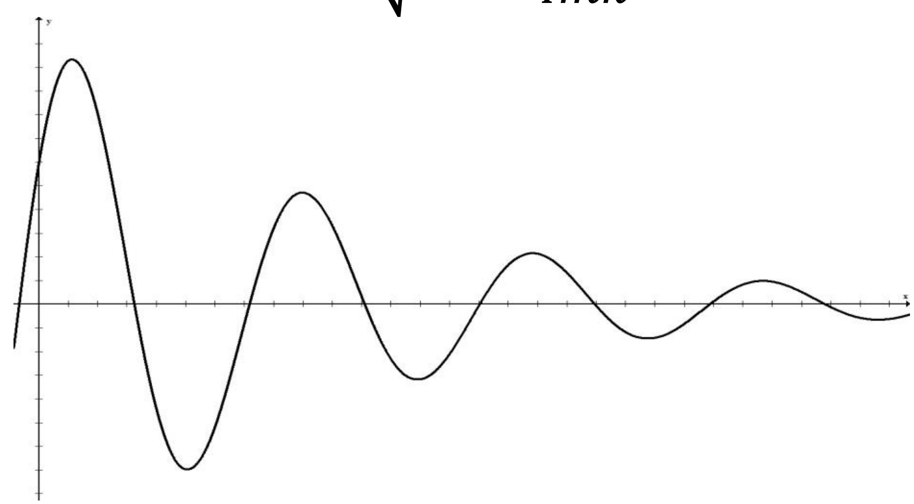
$$\lambda = -\frac{\gamma}{2m}$$

$$\mu = \frac{1}{2m}\sqrt{4mk - \gamma^2}$$

$$= \sqrt{\frac{k}{m} - \frac{\gamma^2}{4m^2}} = \sqrt{\frac{k}{m}}\sqrt{1 - \frac{\gamma^2}{4mk}}$$

= **quasi**-frequency

Note: $\mu = \omega_0\sqrt{1 - \frac{\gamma^2}{4mk}}$



3.8 External forcing

We will discuss a wave forcing function of the form

$$F(t) = F_0 \cos(\omega t)$$

$$F_0 = \text{external force amplitude}$$

$$\omega = \text{external force frequency}$$

Case 3: $\boldsymbol{\gamma = 0}$

$$mu'' + ku = F_0 \cos(\omega t)$$

Homogeneous solution

$$c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$$

General solution

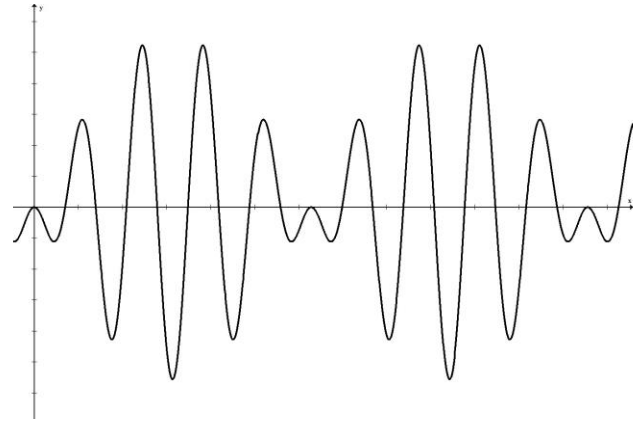
$$u(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t) + u_p(t)$$

Particular solution?

3a: If $\omega \neq \omega_0$, then use

$$Y(t) = A \cos(\omega t) + B \sin(\omega t)$$

$$= \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos(\omega t)$$



Aside: The picture above is a soln to

u'' + 16u = cos(5t)

3b: If $\omega = \omega_0$, then use

$$Y(t) = At \cos(\omega t) + Bt \sin(\omega t)$$

$$= \frac{F_0}{2m\omega_0} t \sin(\omega t)$$